Homework Exercises on Linear Systems

LS1. Consider the following system of linear equations.

$$x + 2y = 8$$

$$3x + 4y = 12$$

a) Write this system of equations in its corresponding matrix form. Write the augmented matrix associated with this system of linear equations.

b) Using Gauss-Jordan elimination, determine the solution to this system. [Write your solution in vector form.]

c) On a single set of coordinate axes, plot the graphs of the two linear equations above. Then label your solution in b) on the graph. To what does your solution correspond graphically?

LS2. Consider the following system of linear equations.

$$x_1 + 4x_2 + x_3 + x_4 = 2$$

$$2x_1 + 6x_2 + 2x_4 = 0$$

a) Write the augmented matrix associated with this system of equations.

b) Use Gauss-Jordan elimination (also called Gauss-Jordan row reduction) to find the reduced row echelon form of the augmented matrix. [Indicate clearly the elementary row operations that you use. This information will needed in part e.]

c) Write down the system of linear equations whose corresponding augmented matrix is the reduced echelon form above.

d) Find the general solution to this system of equations. [Express the general solution in vector form.]

e) For each elementary row operation used in b), write down the associated elementary matrix.

(More questions on next page.)

LS3. Here is an interesting system of linear equations.

$$x_1 + 3x_2 + x_3 = 1$$
$$2x_1 + 5x_2 + x_4 = 3$$
$$x_2 + x_5 = 3$$

a) Write the augmented matrix associated with this system of equations.

(Observe: Why is it interesting? This particular system is considered *reduced* with respect to the variables x_3, x_4, and x_5. What this means is that the columns correponding to the variables x_3, x_4, and x_5 are the columns of an identity matrix.)

b) Now use Gauss-Jordan elimination to find the reduced row echelon form of the augmented matrix.

(Observe: The answer for b) is considered to be reduced with respect to the variables x_1, x_2, and x_3, because the columns correponding to the variables x_1, x_2, and x_3 are the columns of an identity matrix.)

c) Write down the system of linear equations whose corresponding augmented matrix is the reduced row echelon form above.

d) Find the general solution to this system of equations. [Express the general solution in vector form.]

e) Using your answer in d), find the three specific solutions given by the following choices of the free (a.k.a. non-basic) variables.
(i) $x_4 = 0$, $x_5 = 0$
(ii) $x_4 = 1$, $x_5 = 0$
(iii) $x_4 = 0$, $x_5 = 1$

f) Check that your solutions in e) satisfy the original system of linear equations. Check that they also satisfy the system of linear equations you gave in c). [If this reveals any errors, then re-examine your work in the previous steps.]

(Recall and Observe: By the nature of the elementary row operations, the solution set of the original system of equations is the same as the solution set of the transformed system arising from elementary row operations. But the transformed system may allow one to make observations about the solution set that were not so apparent from the original system.)

g) Is it possible to take your augmented matrix from part b) and reduce it with respect to the variables x_1, x_2, and x_4? If so, what matrix do you get? What is the corresponding set of equations? What is the general solution in vector form that naturally arises from this set of equations?