MATH 497 - Additional Questions for Homework on Section 3.3.

Suppose that $S_{1}, S_{2}$, and $S_{3}$ are risky securities with the following information

| Security | Expected Return $\left(\mu_{j}\right)$ | Risk $\left(\sigma_{j}\right)$ |
| :---: | :---: | :---: |
| $S_{1}$ | 0.05 | 0.03 |
| $S_{2}$ | 0.10 | 0.12 |
| $S_{3}$ | 0.08 | 0.06 |


| Security Pair | Correlation Coeff. $\left(\rho_{i j}\right)$ |
| :---: | :---: |
| $S_{1}-S_{2}$ | -0.2 |
| $S_{1}-S_{3}$ | 0.5 |
| $S_{2}-S_{3}$ | 0.2 |

A. Calculate the expected return and risk for the following portfolios
(i) the portfolio consisting of of $70 \%$ of $S_{1}$ and $30 \%$ of $S_{2}$ by weight
(ii) the portfolio evenly divided between $S_{1}, S_{2}$, and $S_{3}$ by weight
(iii) the portfolio with weightings $150 \%$ security $S_{1}, 50 \%$ security $S_{2}$ and $-100 \%$ security $S_{3}$
B. Calculate the weights of the minimum variance portfolio for this set of securities. Calculate the expected return and risk of the minimum variance portfolio. [Note: You may wish to use Mathematica to calculate the various matrix inverses.]
C. Calculate the weights and risk of the portfolio on the minimum variance line that have expected return
(i) $\mu=0.02$
(ii) $\mu=0.04$
(iii) $\mu=0.06$
(iv) $\mu=0.08$
(v) $\mu=0.10$
D. Suppose that one has access to a risk-free return of $2 \%$.
(i) Calculate the Sharpe ratio of each security $S_{1}, S_{2}$, and $S_{3}$.
(ii) Calculate the Sharpe ratio of each portfolio in problem A .
(iii) Calculate the weights of the market portfolio for this set of securities. Calculate the expected return and risk of the market portfolio. [Note: You may wish to use Mathematica to calculate the various matrix inverses.]
E. Plot the risk-return profile (on a $\sigma \mu$ plane) of each of the securities $S_{1}, S_{2}$, and $S_{3}$, the portfolios in problem B and C, and the market portfolio. Sketch the graph of the minimum variance line.

