CMPSC 360 - Additional Questions for Homework on Section 5.3.

A. Consider the set of bit strings S defined as follows

- The empty string λ is in S
- If $t \in S$ then $01t \in S$ and $0t1 \in S$

(a) Construct all the bit strings that would be formed by applying the recursive step at most three times.

(b) Prove that every bit string in S has an equal number of 0's and 1's.

B. Consider the set of strings P (using characters "(" and ")") defined as follows

- The empty string λ is in P
- If $q, r \in P$ then the concatenation $qr \in P$
- If $q \in P$ then $(q) \in P$
  [Note: Treat the left parenthesis "(" and right parenthesis ")" simply as characters in a string.]

(a) Construct all the strings that would be formed by applying recursive steps at most three times.

(b) Prove that every string in P has an equal number of left parentheses and right parentheses.

C. Consider the set of integers T defined as follows

- $1 \in T$
- If $n \in T$ then $n - 4 \in T$
- If $n \in T$ then $2n + 1 \in T$

(a) Construct all the integers that would be formed by applying recursive steps at most three times.

(b) Prove that every integer in T is odd.

D. Consider the set of strings U (using characters from the set $\{\text{A}, \text{B}\}$) defined as follows

- $\text{A} \in U$
- If $t \in U$ then $\text{AB}t \in U$

(a) Construct all the strings that would be formed by applying recursive steps at most four times.

(b) Prove that every string in U ends with the character A.

(c) Prove that for every string in U the total number of A's present is greater than the total number of B's present.

E. Consider the definition of full binary trees defined as follows

- The single vertex r (designated as a root) is a full binary tree.
- If T_1 and T_2 are full binary trees, then there is a full binary tree $T_1 \cdot T_2$ consisting of a root r together with edges connecting to T_1 (as its left subtree) and T_2 (as its right subtree).

Prove for any full binary tree T that the number of vertices that are left children equals the number of vertices that are right children.