Additional Questions for Homework on Section 5.4 and Section 7.1.

A. Consider the sine intergral function $\mathrm{Si}(x)$ defined by

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\, dt.$$

Calculate the following derivatives.

(i) $\frac{d}{dx}\,\mathrm{Si}(3x)$ (ii) $\frac{d}{dx}(x^2\,\mathrm{Si}(x))$ (iii) $\frac{d}{dx}(\mathrm{Si}(x))^3$ (iv) $\frac{d}{dx}\,\mathrm{Si}(1/x)$

B. Consider the error function $\mathrm{erf}(x)$ that is defined by

$$\mathrm{erf}(x) = \int_0^x e^{-t^2}\, dt.$$

Calculate the following derivatives.
(i) $\frac{d}{dx}\,\mathrm{erf}(x/2)$ (ii) $\frac{d}{dx}(x\,\mathrm{erf}(x))$ (iii) $\frac{d}{dx}(4\,\mathrm{erf}(x)+2)$

C. Consider the Fresnel S function $S(x)$ that is defined by

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)\, dt.$$

Calculate the following derivatives (also use the function definitions from the previous two problems).
(i) $\frac{d}{dx}S(x)$ (ii) $\frac{d}{dx}\,(S(x)\,\mathrm{erf}(x))$ (iii) $\frac{d}{dx}\,\frac{\mathrm{Si}(x)}{S(x)}$

D. Based on the integral construction of the natural logarithm,

(a) draw the region whose area corresponds to $\ln 1.5$.

(b) Based on your answer to (a) explain why one can graphically conclude that $1/3 \le \ln 1.5 \le 1/2$.

(c) In fact one can do even better. See if you can explain why $1/3 \le \ln 1.5 \le 5/12$. (Hint: Think trapezoids.)

(d) Draw the region whose area corresponds to $\ln 3$.

(e) Based on your answer to (a), explain graphically why $\ln 3 \le 2$.

(f) Determine a better upper bound for $\ln 3$ using graphical means and support your answer.